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INTERACTION OF AN UNSTEADY TWO-PHASE JET WITH A LAYER OF A DISPERSE MEDIUM

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Discharge of a two-phase jet from a cylindrical channel into a bounded layer of a disperse medium is numerically simulated using the equations of the mechanics of heterogeneous media with allowance for the differences in velocity, temperature, and phase stresses. The effect of separation of the gas phase from the disperse phase in the layer is revealed and verified experimentally. A comparison with a similar process of gas discharge at equal initial pressures shows that in the interaction with the disperse layer, the two-phase flow has a longer momentum and direction.

Introduction. In recent years, two-phase flows have found wide technical application in both traditional areas (jet transport and fire extinguishing [1, 2]) and new fields (actuation of the final control elements of drives that do not require special seal [3, 4]).

The goal of the present paper is to study the discharge of a two-phase medium with a nearly close packing at the initial moment into a bounded layer of a disperse medium. We consider soft media such as semiconsolidate sands and soils occurring on the surface.

Formulation of the Problem. We consider a two-phase medium within the framework of the model of a gas-saturated porous medium at a rather high concentration (one of the possible close packings) using the well-known assumptions [5]. In the rarefied state, the medium is treated as a gas–pseudogas of particles [4]. The motion of the two-phase mixture can be described by the following equations of conservation of energy, mass, and momentum, which are unified over the entire region of integration:

$$\frac{\partial \rho_i}{\partial t} + \nabla \cdot \rho_i \boldsymbol{v}_i = 0,$$

$$\frac{\partial \rho_1 \boldsymbol{v}_1}{\partial t} + \nabla \rho_1 (\boldsymbol{v}_1 \boldsymbol{v}_1) + \beta_1 \nabla p + (1 - \beta_2)(\nabla p_d - \nabla \sigma_f) = -\beta_3 \boldsymbol{F}_\mu + \beta_3 \rho_1 \boldsymbol{g} + (1 - \beta_2)(\rho_1 + \rho_2)\boldsymbol{g},$$

$$\frac{\partial \rho_2 \boldsymbol{v}_2}{\partial t} + \nabla \rho_2 (\boldsymbol{v}_2 \boldsymbol{v}_2) + (1 - \beta_1) \nabla p + \beta_2 (\nabla p_d - \nabla \sigma_f) = \beta_3 \boldsymbol{F}_\mu - \beta_3 \rho_1 \boldsymbol{g} + \beta_2 (\rho_1 + \rho_2)\boldsymbol{g},$$

$$\frac{\partial \rho_2 u_2}{\partial t} + \nabla \cdot \rho_2 u_2 \boldsymbol{v}_2 = Q + H_{sh},$$

$$\frac{\partial \rho_2 k_2}{\partial t} + \nabla \cdot \rho_2 k_2 \boldsymbol{v}_2 + p_d \nabla \cdot \boldsymbol{v}_2 = H_\mu^{(t)} - H_{sh} - H_\mu, \tag{1}$$

$$\frac{\partial}{\partial t}(\rho_1 E_1 + \rho_2 E_2) + \nabla \cdot [\rho_1 E_1 \boldsymbol{v}_1 + \rho_2 E_2 \boldsymbol{v}_2 + p(\alpha_1 \boldsymbol{v}_1 + \alpha_2 \boldsymbol{v}_2) + p_d \boldsymbol{v}_2 - \sigma_f \boldsymbol{v}_2] = \rho_1 \boldsymbol{g} \cdot \boldsymbol{v}_1 + \rho_2 \boldsymbol{g} \cdot \boldsymbol{v}_2,$$

$$\rho_i = \rho_i^0 \alpha_i \quad (i = 1, 2), \qquad E_1 = u_1 + v_1^2/2, \quad E_2 = u_2 + k_2 + v_2^2/2,$$

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$$\beta_1 = \frac{\alpha_1(2 + \chi_m \rho_1^0/\rho_2^0)}{2 + \chi_m(\alpha_2 + \alpha_1\rho_1^0/\rho_2^0)}, \quad \beta_2 = \frac{2 + \chi_m \alpha_2}{2 + \chi_m(\alpha_2 + \alpha_1\rho_1^0/\rho_2^0)}, \quad \beta_3 = \frac{2}{2 + \chi_m(\alpha_2 + \alpha_1\rho_1^0/\rho_2^0)}.$$

Hereinafter, the subscripts 1 and 2 refer to the parameters of the carrier and disperse phases, respectively, the superscript 0 refers to true values of density, ∇ is the Hamiltonian, α_i, ρ_i, $\boldsymbol{v}_i$, E_i, and u_i are the volume fraction, normalized density, vector velocity, toal and internal energy of a unit mass of the ith phase, respectively, p, p_d, σ_f, k_2, and $\boldsymbol{g}$ are the gas pressure, effective pressure due to random particle motion, the tensor of effective stresses, pulsation energy of a unit mass of the disperse phase, and the free-fall acceleration vector, respectively, $\boldsymbol{F}_\mu$, Q, $H_\mu^{(t)}$, H_μ, and H_{sh} are the viscous component of the interphase interaction force, the rate of heat exchange between the gas and the particles, rate of generation of the energy of random particle motion due to vortex flow, viscous dissipation, and the dissipation due to inelastic collisions, respectively, χ_m is a coefficient that takes into account the effect of the nonsingleness and nonspherical shape of the particles on the attached-mass force ($\chi_m = 1$ for spherical particles), and t is time.

To close system (1), we use the equations of state for an ideal, calorifically perfect gas

$$p = (\gamma_1 - 1)\rho_1^0 u_1, \qquad u_1 = c_v T_1$$

and incompressible solid particles

$$u_2 = c_2 T_2, \qquad \{\gamma_1,\, c_v,\, c_2,\, \rho_2^0\} \equiv \text{const},$$

the equation of interphase bonds in the form of the generalized Hooke's law for a porous saturated medium [5]

$$\sigma_f^{kl} = \alpha_2[\lambda_f^{(\sigma)} \varepsilon_2^{mm} \delta^{kl} + 2\mu_f^{(\sigma)} \varepsilon_2^{kl} + \nu_f^{(\sigma)} p \delta^{kl}] \qquad \text{for} \quad \alpha_2 \geqslant \bar{\alpha}_2,$$

and the equation of a pseudogas of particles [4]

$$p_d = (2/3)G(\alpha_2)\rho_2 k_2 \quad \text{for} \quad \alpha_2 < \bar{\alpha}_2, \qquad G(\alpha_2) = 1/[1 - (\alpha_2/\bar{\alpha}_2)^{1/3}].$$

Here T_1 and T_2 are the temperature of the carrier phase and the particles, γ_1 and c_v are the adiabatic exponent and the specific heat of the gas with constant volume, c_2 is the specific heat of the particles, ε_2 is the macrostrain tensor for the second phase, $\lambda_f^{(\sigma)}$, $\mu_f^{(\sigma)}$, and $\nu_f^{(\sigma)}$ are the characteristics of the porous elastic medium, determined according to [5, 6], $G(\alpha_2)$ is the Enskog correction function, which describes the increase in the number of collisions in the concentrated gas as compared to the dilute gas, and $\bar{\alpha}_2$ is the particle concentration corresponding to close packing.

The interphase friction intensity $\boldsymbol{F}_\mu$ and the heat-transfer rate Q are specified by the well-known empirical relations, tested for the problems of this class [5, 7, 8]. The averaged interfacial and interparticle energy exchange $H_\mu^{(t)}$, H_μ, and H_{sh} are used in the form given in [4].

We consider the following problems. At the initial instant, a cylindrical channel having a length $L = 0.8$ m and a diameter $D = 0.1$ m is filled with a high-pressure gas at rest (the first problem) and particles (the second problem) with a nearly close packing (bulk state). The high-pressure chamber is separated from the ambient medium by a diaphragm. The channel is placed at a depth $H = 0.4$ m from the surface under a layer of a granular medium. At time $t = 0$, the diaphragm is removed. We need to study the flow at $t > 0$.

The problems were solved for the following initial data: $p_0 = 1$ MPa, $p_a = 0.1$ MPa, $T_{i0} = T_{ia} = 293$ K, $\alpha_{20} = \alpha_{2a} = 0.5$, $\gamma_1 = 1.4$, $c_v = 716$ m^2/(sec$^2 \cdot$K), $c_2 = 710$ m^2/(sec$^2 \cdot$K), particle diameter is $d = 100$ μm, $\rho_2^0 = 2600$ kg/m^3, $\bar{\alpha}_2 = 0.63$, the subscripts 0 and a refer to the parameters in the high-pressure chamber and outside it, respectively. In the second problem, it is assumed that $\alpha_{20} = 0$.

A solution of the problems is obtained by the method of [9], in which all source components ($\boldsymbol{F}_\mu$, Q, $H_\mu^{(t)}$, etc.) are taken into account implicitly in the first step. This allows us to increase substantially the stability factor of the difference scheme. For the two-phase flows considered, the allowable step in time is 3–4 times (for certain regimes, an order of magnitude) larger than for the case of explicit specification of interphase interactions. This question is considered in greater detail in [10], where it is shown that rigidity is a fundamental property for a wide class of problems of the wave dynamics of two-velocity, two-temperature media. Taking into account this property, we can distinguish schemes of advanced stability.

The boundary conditions for the problems are the nonpenetration condition at the walls and the initial conditions at infinity. The calculations were performed using a through method without distinguishing discontinuities in the cylindrical (with axial symmetry) coordinate system on a uniform 150×50 grid. Calculation results show that in order to diminish oscillations at $\alpha_2 > \bar{\alpha}_2$, it is appropriate to supplement the effective stresses in the porous medium with artificial viscous stresses, for example, of the Landshoff type.

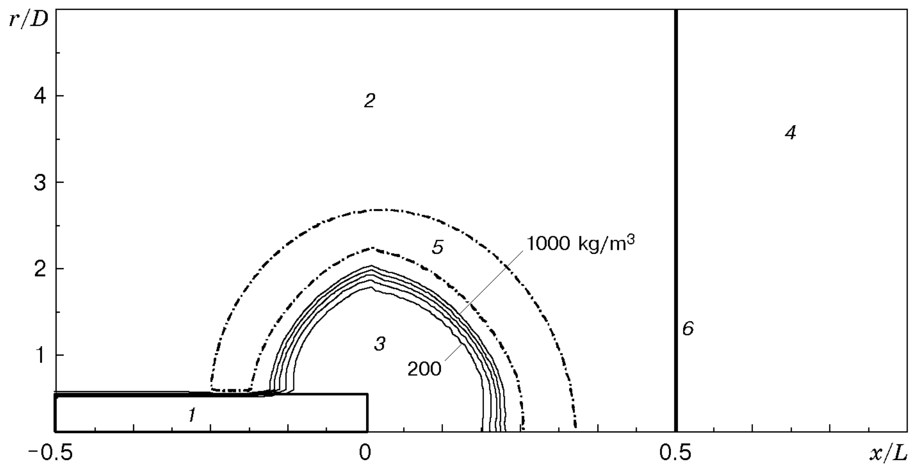

Fig. 1. Calculations of gas discharge for time $t = 0.01$ sec ignoring effective stresses: 1) channel; 2) disperse medium; 3) cavity; 4) gas; 5) layer of close-packed particles ($\alpha_2 \geqslant \bar{\alpha}_2 = 0.63$); 6) interface; solid curves refer to $\rho_2 = 200$, 400, 600, 800, and 1000 kg/m^3.

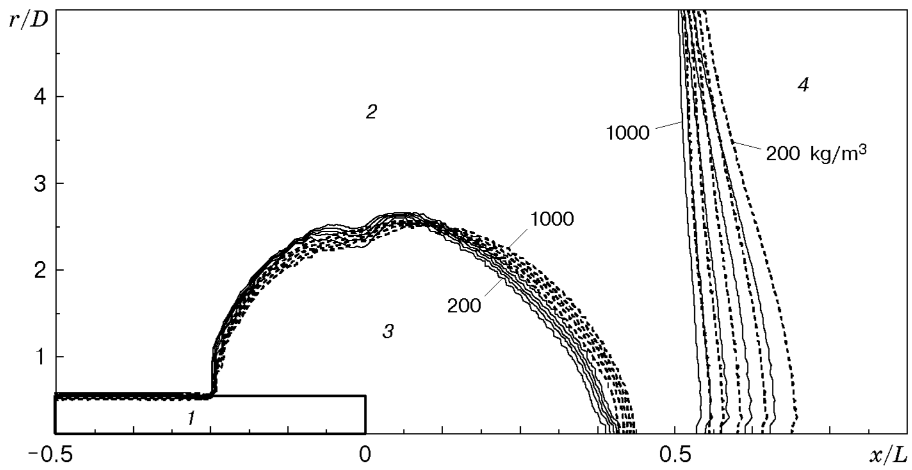

Fig. 2. Calculations of gas discharge for time $t = 0.05$ sec ignoring (dashed curves) and allowing (solid curves) for effective stresses (notation same as in Fig. 1).

Results of Numerical Simulation. In the present paper, we consider the case of $\nu_f^{(\sigma)} \ll 1$. According to the classification given in [5], such disperse media are called "soft media" (sands of bulk density and soils adjacent to the surface). As shown in [5], in the case of soft media, the effective stresses σ_f can often be neglected.

Test calculations of the first problem were performed under the assumption of no particle interaction. Figure 1 shows calculation results for time $t = 0.01$ sec. The x axis corresponds to the symmetry axis and the y axis is directed along the radius. During gas discharge from the channel 1 into the disperse layer 2, an almost spherical cavity 3 develops. In the vicinity of the cavity, a layer of close-packed particles 5 forms (the dot-and-dashed curve in Fig. 1 bounds the region of particle concentration in the layer $\alpha_2 \geqslant \bar{\alpha}_2 = 0.63$). The gas is filtered through the porous medium and, by means of interfacial friction, causes particle motion in the disperse medium. Then, at $t = 0.02$ sec, the particle concentration is $\alpha_2 < \bar{\alpha}_2$ over the entire calculation region. The flow pattern at $t = 0.05$ sec is shown in Fig. 2. Here the density fields of the disperse phase are given [dashed isolines are obtained ignoring the effective stresses, and solid isolines are obtained within the framework of model (1)]. A comparison of the results obtained shows that interphase interaction is the determining flow mechanism in a soft disperse medium. However, the use of the model ignoring effective stresses in the layer is quite incorrect. First, at the initial stage of motion, the particle concentration in the layer $\alpha_2 > 0.8$ exceeds the limiting tetrahedral packing $(\bar{\alpha}_2)_{\text{tet}} \approx 0.74$; second, an increase in the initial pressure in the channel yields $\alpha_2 > 1$, i.e., calculations become unfeasible.

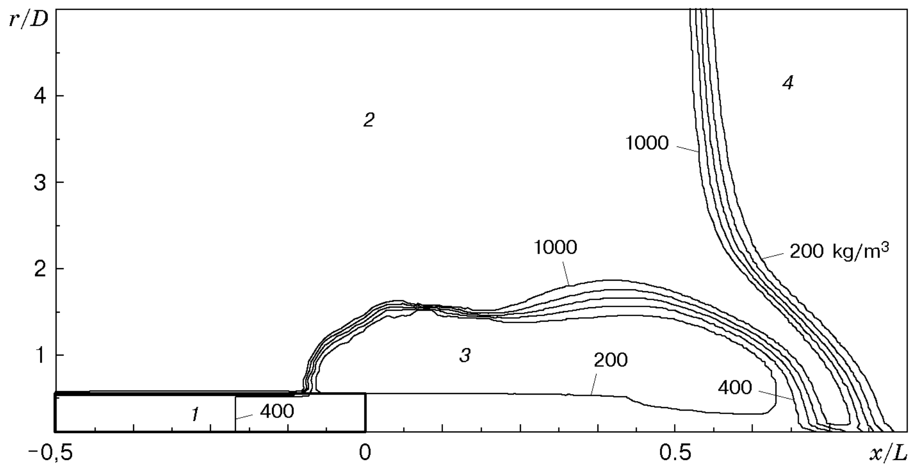

Fig. 3. Calculations of discharge of the two-phase mixture for time $t = 0.05$ sec (notation same as in Fig. 1).

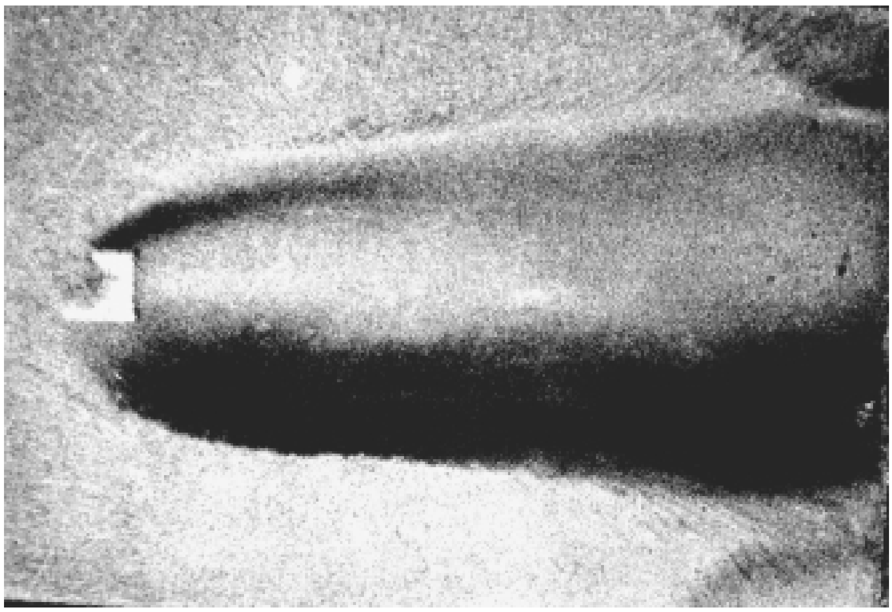

Fig. 4. Photograph of a gas-disperse flow discharging into a layer of quartz sand.

Calculations of the discharge of the gas-disperse mixture from the cylindrical channel into the bounded disperse layer were performed within the framework of the model of a two-phase medium (1) and the problem formulated. After rupture of the diaphragm, the two-phase mixture begins to escape into the layer, which is accompanied by the formation of a cavity. With time the mixture phases separate from each other (Fig. 3). Figure 3 shows calculation results for $t = 0.05$ sec. It can be seen that with increase in radius, a region 3 with a predominant volume fraction of the gas forms. Calculations were performed using the through method, and the cavity boundary was "spread" over one or two to several cells. On the periphery of the two-phase flow, the value of α_2 is less than 1% of the value of $\bar{\alpha}_2$ corresponding to the close packing of particles.

The observed effect of phase separation is confirmed experimentally. Figure 4 shows a photograph of a gas-disperse medium discharging into a layer of quartz sand. Light zones correspond to higher particle density. In the center, one can see a jet surrounded by a dark zone with low particle concentration. The flow is asymmetric, in particular, because of insufficiently quick removal of the diaphragm.

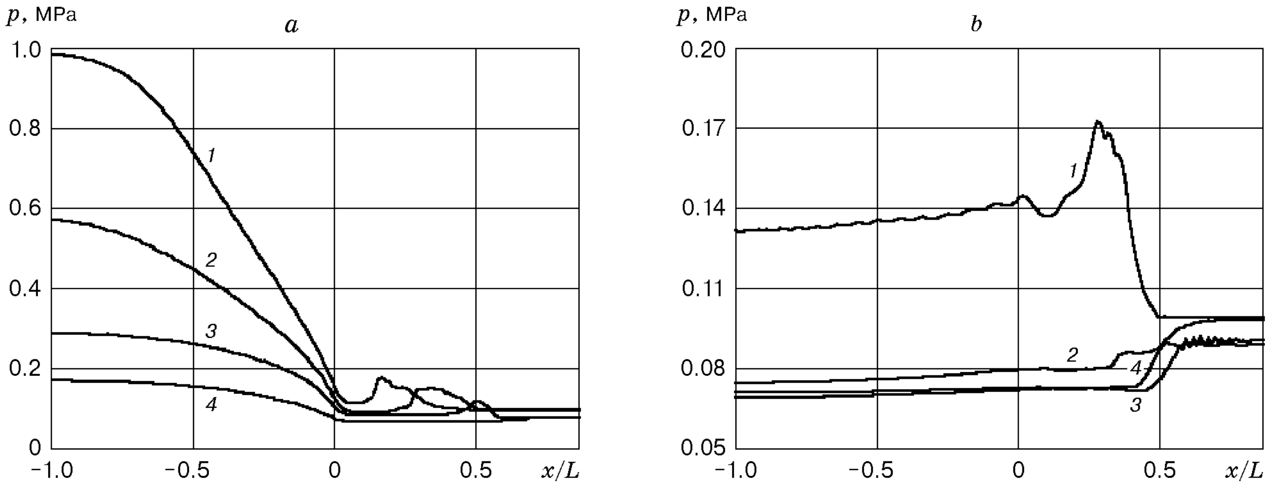

Fig. 5. Gas-pressure distribution along the symmetry axis during discharge of two-phase (a) and one-phase (b) flows for $t = 0.0125$ (1), 0.0250 (2), 0.0375 (3), and 0.0500 sec (4).

A comparison of the interaction of the gas and two-phase flows with the layer shows that in the first problem (see Fig. 2), the cavity grows faster and has a larger volume. An analysis of the shape of the layer surface and the vector velocity field indicates a more "concentrated" action of the gas-disperse flow. We note that at the same initial gas pressure in the channel, the initial value of the internal gas energy for the second problem is smaller by a factor of $1/\alpha_1$ than that for the first problem. However, according to the calculation data, in a region having a size of the order of the channel diameter at $t = 0.05$ sec, the velocity and momentum of the layer are several times higher in the case of the gas-disperse flow than in the case of the gas flow.

Additional data on the process can be obtained from an analysis of the gas-pressure distribution along the symmetry axis during discharge of the two-phase (Fig. 5a) and one-phase (Fig. 5b) flows (curves 1–4 refer to $t = 0.0125$, 0.0250, 0.0375, and 0.0500 sec, respectively). Interaction of the two-phase flow with the layer is of a wavy character and results in the formation of characteristic zones. In contrast, the gas discharge is quasisteady-state. We compare the characteristic times of the processes: the duration of passage of an infinitesimal perturbation from the edge to the bottom of the channel $\tau_j = L/a_j$, $j = 1, 2$ [a_j is the speed of sound in the two-phase medium [1] ($j = 1$) or in the gas ($j = 2$)] and the observation time. With allowance for the initial data given above, the characteristic discharge time of the two-phase flow τ_1 is comparable to the observation time, whereas for the gas, the value of τ_2 is an order of magnitude smaller.

In the case of two-phase flow discharge (see Fig. 5a), we can distinguish the following characteristic zones. In the channel ($0 \leqslant z \leqslant L = 0.8$ m), the gas pressure decreases in the incident and reflected rarefaction waves, and the pressure difference relative to atmospheric (initial) pressure remains considerably longer than that in the case of gas discharge (see Fig. 5b). At larger x/L, there is a "ledge" in the cavity region (see Fig. 5a). This can be explained by the fast pressure equalization in the gas-disperse flow due to the smaller characteristic size in comparison with the channel length. Since the granular layer is permeable for the gas, the gas is filtered through the layer, which results in pressure growth. Finally, at the times considered, the gas pressure above the layer is close to the initial value.

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